EMC Modeling of an Industrial Variable Speed Drive With an Adapted PEEC Method

Vincent Ardon¹,², Jérémie Aime¹,³, Olivier Chadebec¹, Edith Clavel¹, Jean-Michel Guichon¹, and Enrico Vialardi¹

¹Grenoble Electrical Engineering Laboratory, Université de Grenoble (Grenoble-INP, UJF, UMR CNRS 5269), 38402 Grenoble, France
²Technological Department-Software, Cedrat, 38246 Meylan, France
³Schneider and Toshiba Inverter Europe, 27120 Pacy sur Eure, France

This paper presents an adapted partial element equivalent circuit (PEEC)-based methodology applied to the modeling of interconnections of power electronics devices. Although this method is already well known, the originality of this work is its use to model a device presenting an industrial complexity. To make possible this modeling, two adapted integral methods, based on two different meshings, are presented. They are dedicated respectively to the computation of parasitic inductances and capacitances and lead to an equivalent circuit of the system. From a time-domain simulation of this circuit, current and voltage sources can be extracted and used to compute the radiated near magnetic field. This approach has been applied to model a real industrial static converter via system couplings, a variable speed drive. Good agreements have been obtained between simulated and measured results on conducted and emitted electromagnetic analysis.

Index Terms—Electromagnetic compatibility, fast multipole method, parasitic capacitances, parasitic elements, partial element equivalent circuit (PEEC), power electronics, power interconnections.

I. INTRODUCTION

NOWADAYS, power-electronic designers need more and more accurate modeling tools able to simulate complex geometries, in order to save money and time on tests. Recent works and researches trends [1] have proven that this can be achieved by means of a system coupling approach and the use of a complete electrical equivalent circuit of the device. This circuit approach is interesting because it avoids iterations between different modeling software: in the equivalent circuit, physical and electrical behaviors are modeled together. For example, to analyze the electromagnetic compatibility (EMC) performances with respect to standards which are becoming more and more stringent, it is necessary to model common and differential mode currents flowing into the power interconnections, and consequently, common and differential mode loops—via parasitic capacitances—appear and emit magnetic field into the device. The constraints within power-electronics structures are the following:

- The space occupied by air medium is abundant and greater than the volume of conductors.
- Geometries are often complex, compact, irregular, and composed by multilayers of conductors separated by dielectric substrates.

The problem can be better formalized by considering the integral form of the Maxwell’s equations and by assuming:

— quasi-static conditions;
— only surface location for the free-charges \( \rho \);
— currents \( I \) are assumed uniform and constant in each volume element of conductors,
— vacuum permeability \( \mu_0 \) surrounding the objects;
— a homogeneous medium of permittivity \( \varepsilon = \varepsilon_r \cdot \varepsilon_0 \);
— neglected losses in dielectric materials.

In such conditions, the external applied electric field \( E \) applied at a point \( r \) of a conductor at the pulsation \( \omega \) can be written as (1) where \( J \) is current density, \( \sigma \) the material conductivity, \( A \) the magnetic vector potential, \( \varphi \) the electric potential and \( G \)

\[
E(r) = \frac{J(r)}{\sigma} + j\omega A(r) + \nabla \varphi(r)
\]

\[
= \frac{J(r)}{\sigma} + j\omega \mu_0 \int_V \int_S G(r, r')J(r')dS
\]

\[
+ \frac{1}{\varepsilon} \int_S G(r, r')\rho(r')dS \quad (1)
\]

The three parts of (1) are respectively due to resistive, inductive and capacitive behaviors, which represent the main effects that have to be taken into account in the power-interconnection modeling.

To model a complex industrial device, the finite element method [2] would be little adapted regarding the difficulty to properly mesh volume air regions and conductors to accurately take into account skin and proximity effects. The use of approximated analytical formulas or multiconductor transmission
line theory [3] is also to banish because they are not accurate enough and the medium is not always homogeneous. On the contrary, the partial element equivalent circuit (PEEC) method (i.e., extraction of equivalent circuit components thanks to integral approach) is known to be well adapted to model complex geometries with an important surrounded air region [1]–[4]. However, the classical PEEC method which is widely used at high frequencies (antennas, RF devices, etc.), would be very memory-consuming and not adapted to the consideration of skin effects, as it is based on dual inductive and capacitive meshings. This method would require the computation of a parasitic capacitance between each surface mesh element of conductors, and to take into account accurately the skin effects, the meshing would need to be refined on the conductor sides (an adapted meshing with two mesh elements in the skin depth). Consequently, the inductive effects would be difficult to model and on the contrary, capacitive effects too detailed depending on the frequency range.

In this paper, an adapted technique that takes advantages from different integral methods to analyze the performances of an industrial variable speed drive is proposed. The modeling strategy is based on the extraction of two series of equivalent electrical parameters and on a scheme for coupling them into a global circuit. In fact, on the one hand, parasitic resistances (R), inductances and mutual-inductances (L-M) are computed from a volume meshing of the conductors. On the other hand, capacitances (C) are extracted from free-charges located on conductor and dielectric interfaces. A non-necessarily conformal surface meshing adapted to the spatial location of free-charges is used in order to limit the number of surface elements and to improve the modeling of side effects.

Those extractions of equivalent parasitic components are presented in the two next sections, whereas in Section IV, the construction of the complete equivalent circuit is detailed. In the last section, the proposed approach is tested by modeling an industrial variable speed drive and some comparisons between simulations and measurements are carried out: the behavior of the harmonic response and the emitted near magnetic field are analyzed and compared to measurements.

II. EXTRACTION OF PARASITIC EQUIVALENT RESISTANCES AND INDUCTANCES

This section details methods and meshing techniques used for the computation of parasitic elements which model the resistive and the inductive behaviors of an interconnection structure. The volume of the conductor is meshed into parallelepiped elements where the current density is assumed uniform. The main advantage of our approach is the fact that the meshing depends on a chosen frequency. To properly model skin and proximity effects, conductors must be considered either unidirectional (thin or long tracks) or bidirectional (large tracks and ground planes), depending on the directions of the flowing current. The conductors belonging to the first type are meshed in the skin depth (at least two elements in the skin depth) but not in the length (gain in number of elements), whereas the bidirectional ones are discretized in two directions.

To compute a parasitic resistance $R_i$ in each volume element $V_i$ of length $l_i$, section $S_i$, and resistivity $\rho$, the following analytical formula is used:

$$R_i = \frac{\rho}{S_i} l_i. \quad (2)$$

Each mesh element $V_i$ presents also a self inductance $L_i$ depending from its section $S_i$ and its length $l_i$ (3). Moreover, each couple of nonperpendicular elements is also characterized by a mutual inductance which is computed by integrating the magnetic vector potential created by one volume element $V_i$ on the other $V_j$ and with the unit vectors $\mathbf{u}_i$ and $\mathbf{u}_j$ which define the direction of the flowing current

$${L_i}_j = \frac{\mu_0}{4\pi} \iiint_{V_i} \iiint_{V_j} G(r_i, r_j) dV_i dV_j, \quad (3)$$

Because of the parallelepiped shape of the elements the double integral in $L_i$ can be expressed in an analytical form and easily computed. The mutual inductances $M_{ij}$ are computed thanks to a analytical/numerical integration technique (an analytical expression for the first integral is used, the second one being computed thanks to an adaptive gauss point integration ensuring a good accuracy). It is worth to notice that the mesh elements are not necessarily parallel. All values of $L_i$ and $M_{ij}$ can then be organized in a dense and square matrix $[L-M]$ whose size is equal to the number of mesh elements [5].

To illustrate our approach let us consider a 3-phase busbar system, where each phase is constituted by six parallel bars (Fig. 1, on the right) [6]. The distribution of the current density along three bars of the same phase is computed by means of the presented PEEC approach and a FEM tool. In the PEEC approach, the bars are not meshed in their length, but only in the cross section: 15 elements along the $z$ axis and 4 along the $x$ axis whereas in the FEM approach, air and bars are meshed with a total number of mesh elements around 600 000. So the gap between degrees of freedom number of the two methods is very different.

However, the results in Fig. 1 show that the two numerical methods are very closed, confirming that this simple PEEC meshing used to extracted parasitic resistances and inductances is efficient to model the electrical behavior of a device with only a few number of mesh elements. But to take into account the electrical couplings like the common mode loops at higher frequencies, capacitive effects have to be added to this inductive approach.

III. PARASITIC CAPACITANCES

A. Free-Charges Computation by a Full Interaction Method

According to the geometrical dimensions and frequency range considered, the propagation phenomena can be neglected.
Consequently, an electrostatic formulation based on the computation of the surface distribution of the free-charges thanks to a meshing of the conductor and dielectric interfaces may be sufficient to model the capacitive effects. In order to take into account side effects and the spatial variation of surface charges, a mesh with thinner elements than the inductive ones but which is not-necessarily conformal is required. Capacitive elements will be then gathered to compute some equivalent capacitances to be connected to the R-L-M circuit.

Let us consider a surface meshing of the conductor and dielectric interfaces composed of Nc conductor elements and Nd dielectric elements. The electric potential $P_i$ at a conductor-dielectric interface and the normal field $E_i$ at a dielectric-dielectric interface (which is due to the jump of the normal field coefficient) can be written as

$$\begin{align*}
P_i(r_i) &= \sum_{j=1}^{Nc+Nd} \frac{1}{\varepsilon_0} \int_{S_j} \sigma_j \cdot G(r_i, r_j) dS_j \\
E_i(r_i) &= \sum_{j=1}^{Nc+Nd} \frac{1}{\varepsilon_0} \int_{S_j} \sigma_j \cdot G'(r_i, r_j) \cdot \mathbf{n}_j dS_j
\end{align*}$$

(4)

where $r_i$ is a point of the space, $\sigma_j$ the surface charge of the element $j$, $G$ the green function, $G'$ the gradient of $G$, $r'_j$ the charge at the other surface element, and $\mathbf{n}_j$ the outside normal vector. This normal field is null at the dielectric-dielectric interfaces.

An integral method, closely linked to the MoM method [7], namely the building of a matrix structure in a full and square interaction matrix, can be solved to compute the surface charges that are assumed constant on each surface element. Traditionally with the MoM method, the coefficients of the interaction matrix are computed between each mesh center point: this is a 0-order point matching approach. This interaction matrix P/E links the charges $Q_c$ and $Q_d$ for the conductor and dielectric interfaces to the potential $V$ of the conductors [2-8-9]

$$\begin{pmatrix}
P \\
E
\end{pmatrix} \cdot \begin{pmatrix}
Q_c \\
Q_d
\end{pmatrix} = \begin{pmatrix}
V \\
0
\end{pmatrix}$$

(5)

where the coefficients are the potential coefficients $P_{ki}$ and the normal field coefficients $E_{kj}$ due to unit charge [9], [10]

$$\begin{align*}
P_{ki} &= 1/\varepsilon_0 \int_{S_j} G'(r_i, r_j) dS_j \\
E_{kj} &= \begin{cases} 
1/\varepsilon_0 \int_{S_j} G'(r_i, r_j) \cdot \mathbf{n}_j dS_j, & i \neq j \\
-\varepsilon_1 + \varepsilon_2 \sum_{S_j} \varepsilon_1, & j = i
\end{cases}
\end{align*}$$

(6)

where the surface of the element $j$, is out the integral because the charge is assumed constant on the surface (0-order). Those coefficients are respectively evaluated between each Nc conductor or Nd dielectric elements and all Nc+Nd elements of the meshing. The surface integrals are numerically computed thanks to the gauss point technique. The computation algorithm of the interaction matrix is totally vectorised in order to avoid double “for” loops and save time. After this computation and to improve the accuracy of the potential coefficient computation $P_{ki}$, analytic formulations are used [10] to correct the diagonal of the P matrix.

Other approaches with different order of integration have been developed and compared to solve the matrix system (5). In a 0-order Galerkin approach, double integrals are used to compute the coefficient of (6) and the second member of the matrix linear system (6) is multiplied by the element surfaces. Potential coefficient $P_{ki}$ are also corrected analytically [11], [12].

The first-order integration with a triangular shape function has also been developed for a Galerkin approach. A first-order point matching method is not adapted because it would try to evaluate the potential at the nodes and the potential is singular at each node of triangular surfaces.

Fig. 2 represents the self-capacitance $C_{11}$ and the integration time of the matrix P/E of one of two parallel plates $(10\times 10 \text{mm}^2, \text{gap } 2 \text{ mm})$ for those three integration techniques. The charges used to compute these capacitances are solved by a LU-decomposition. The capacitance computation from charges will be presented at the end of this section. This figure shows that the Galerkin approach is more accurate than the point matching one. But, as far as the integration time is concerned, the 0 or first-order computation with a Galerkin method is slower than the point matching method because double integrals are computed. A nonconformal meshing can be very interesting to refine easily the areas where the gradient of potential or field are high while saving number of elements. That is why we have chosen to use 0-order point matching.

However, the worst drawback of this integral method is the storage of the full matrix and the integration time which increases in $O(n^2)$. To overcome this issue, the fast multipole method (FMM) appears to be very interesting to enable the modeling of large devices.

**B. Free-Charges Computation by the Fast Multipole Method**

At the origin, the FMM has been developed in order to accelerate the computation of far potential interactions between punctual electric charges [13]. This algorithm is low-memory consuming thanks to the use of a truncated multipole decomposition of interactions. For example the potential far from a set of m charges $Q$ can be written in spherical coordinates as

$$V(r, \theta, \varphi) = \frac{1}{4\pi\varepsilon_0} \sum_{m=-n}^{n} \sum_{n=0}^{\infty} Y_n^m(\theta, \varphi) \cdot P_n^m(\cos \theta) \cdot \rho^{n+1}$$

(8)

$$Y_n^m(\theta, \varphi) = \sqrt{\frac{(n-m)!}{(n+m)!}} \cdot \rho^n \cdot e^{im\varphi}$$

where $P_n^m(\cos \theta)$ are the associated Legendre polynomials.
where $M_{ij}^n$, the multipole depends on the spherical harmonics $Y_{ij}^n$ that are composed of Legendre functions. To use these multipole series efficiently, an octree algorithm is needed to do a hierarchical partitioning of the geometry in different cube levels. This octree drives the type of interactions between each cube depending on the distance between them. To summarize the principle of the method, far interactions are computed by the FMM and the near interactions between adjacent cubes are computed by the full method presented previously.

A second version of the FMM [14], the adaptive multilevel fast multipole method (AMLFMM) is more adapted to a nonuniform distribution of charges. It allows to save memory and to accelerate even more the computation of interactions. However, in our problem, charges are not punctual but linked to surface and the meshing can be nonconformal, but the second part of the FMM theorems ensures an upper bound of the error of the potential, computed by a multipole decomposition, due to a punctual distribution of charges. This maximum error value is not guaranteed with surface charges because the cube partitioning done by the octree algorithm does not deal with these surfaces. Consequently, we have developed a new version, quite similar to [15], that adaptively takes into account the size of the surface mesh element in the octree in order to better evaluate the proximity of the mesh elements and their belonging to a cube at a certain level.

Concerning the far electric field with the FMM, the computational effort, namely the differentiation of the electric potential ($E = -\nabla V$), is relatively weak because only the last coefficient $Y_{ij}^n(\theta, \varphi)/\rho^{n+1}$ in (8) depends on the evaluation point coordinates. Consequently, all the multipole coefficients used to evaluate the electric potential are also used to compute the electric field. This last one converted into Cartesian coordinates by means of a jacobian matrix is multiplied by the unit normal vector of surface.

To solve the problem with the FMM, an iterative solver is necessarily used. Because there is no “true” interaction matrix, a left preconditioned GMRESR(m) algorithm [16] is chosen.

Thanks to the near integration, all the full and square small matrices are inverted and used in the preconditioning [17]. Comparisons of integration and resolution time on the two plates example, plotted in the Fig. 3, shows the dramatic rapidity of the FMM algorithm developed. Moreover, the memory consumption is very low: in the full method, to integer and solve the two linear equation system (5) presented above or with the FMM

$$C_{ij} = \sum_{k=1}^{N} \epsilon_{rk} \frac{Q_{ik}}{r_{ik}} \text{Cond}(i) = 1 \text{ V} \quad \text{Cond}(j) = 0 \text{ V, } i \neq j \quad (9)$$

$$Q_{ci} = C_{ki}V_i + \sum_{j=1}^{N} C_{ij}(V_i - V_j) \quad (10)$$

with

$$\begin{cases} C_{ki} = \sum_{j=1}^{N} C_{ij} \\ C_{kj} = -C_{ji} \quad \text{if } i \neq j \end{cases} \quad (11)$$

where $Q_{ci}$ is the total charge of the conductor $i$ of potential $V_i$ referenced to the ground ($V_{\text{ground}} = 0$ V). A well-known and simple relation (11) permits to obtain the usable capacitance matrix $C'$ from the matrix $C$ [19].

IV. CONSTRUCTION OF THE COMPLETE (R-L-M-C) PARASITIC EQUIVALENT CIRCUIT

The inductive and capacitive parasitic parameters are not directly connected. First the inductive problem is reduced as regards the number of degrees of freedom (Fig. 4). For example, the inductive RLM matrix between the pins $A_1, A_2, B_3$ and $B_2$ of the Fig. 4 is reduced in a matrix 2*2. In the circuit simulator, currents are not solved in each inductive element because it would require too much memory and solving-time at each time step. Let us consider $U_{br}, I_{br}, Z_b$, and $U_b$ respectively the branch voltage and branch current, the impedance matrix composed of the last presented parasitic RLM matrix and eventually other R, L, or C passive elements, and to finish the source voltage. Thus, the full electric system can be written as

$$U_b = Z_b I_b + U_s \quad (11)$$
Actually, a reduced equivalent inductive matrix $Z_L$ is extracted between a set of chosen electric nodes. It is computed at a chosen frequency. Among these nodes, there are those used to connect the parasitic capacitances, the power supply, the load and other electric components of power or command. To reduce the system (11) a matrix $M$ of independent Kirchhoff’s loops is used. $M$ is composed of 0, 1 or $-1$ and describes the connectivity of each inductive element [20]. It is possible to choose $M$ as $M \cdot U_h = 0$. Thus, we can have a set of currents in the independent branches as $I_m = M^T \cdot I_h$. Thus, by multiplying (11) at the left by $M$ and with $Z_m = M \cdot Z_h \cdot M^T$, we can write

$$Z_m \cdot I_m + M \cdot U_h = 0.$$  

After a transformation of $Z_m$ into a partial upper triangular matrix with a Gaussian elimination process, we can obtain the small impedance matrix $Z_A$ composed of equivalent impedances between the chosen electric nodes

$$Z_m = \begin{bmatrix} * & \cdots & \cdots & * \\ 0 & \ddots & \vdots & \vdots \\ \vdots & \ddots & * & * \\ 0 & \cdots & 0 & Z_A \end{bmatrix}.$$  

All the other impedances between other nodes, noted $*$, are not used. From $Z_A$, a macroblock is built with the same number of pins that the size of $Z_A$ and it represents the equivalent impedance at the chosen frequency.

The parasitic capacitances are directly connected to this macroblock. The self capacitive effect of each unidirectional conductor is divided in the two extremities and connected to a reference ground. The coupling capacitive effects with other unidirectional conductors are also divided into their extremities (Fig. 4). To increase the capacitive couplings of very long conductors, it is possible to split those in $k$ subdivisions and to proceed in the same way for the $k$ parts. Thus, capacitive and inductive couplings are modeled by $\Pi$ cells which are more convenient to connect than $T$ cells as extremity nodes of conductors are accessible via the macroblock pins (and not the middle node).

The capacitances between bidirectional conductors (ground planes) and unidirectional conductors are connected in several particular points to accurately take into account proximity effects. Thus, a complete equivalent circuit of interconnections is built. By connecting the rest of the electrical circuit (power, command, etc.) it is possible to simulate the electrical behavior of the power device in a circuit simulator.

V. APPLICATION TO AN INDUSTRIAL VARIABLE SPEED DRIVE

A. Extraction of the (R-L-M-C) Parasitic Equivalent Circuit

These methods presented previously have been used to model a complex real static converter: a marketed variable speed drive.

It is composed of a common mode filter, a power and command module, a mechanical and cabling part. Its function is to drive a three-phase motor depending on a command signal order. The power module is composed of four thin copper-track layers. To analyse this complex device, a system-coupling approach is used.

All of this inductive method described is implemented in InCa3D, a commercially available software [21]. From the inductive matrix, postprocessing modeling gives an accurate description of the current density distribution and losses in the conductors or near magnetic field [2]. A coupling with a circuit solver, a SPICE-like tool, makes possible time-domain analysis by using a reduced R-L-M matrix, the macroblock mentioned before.

The Fig. 5 shows the 1-D (tracks) and 2-D (mechanical parts or ground planes) meshings used to extract the lumped elements: a square resistive-inductive matrix ($6800 \times 6800$ elements). The capacitive meshing contains 48 500 surface elements. 27 conductors are identified and the capacitive matrix is computed with the FMM algorithm and the GMRESR(m) solver. To finish, a system-level software, Saber [22], is used to build the complete PEEC circuit of the interconnections of this device made of three R-L-M macroblocks and all the capacitances which are linked themselves (Fig. 6).

B. Time-Domain Simulations

The supply chain, the load (a three-phase motor) and the command circuit are then added to the previous PEEC circuit. The output DC bus voltage is of 538 V. It is close to the theory (548 V, i.e., gap of 2%). The PEEC circuit has a non-negligible influence on the inverter voltages especially during the commutations of the complementary inverter arms because these last ones have higher frequency signals (Fig. 6). The equivalent circuit of such industrial device is complex. In consequence, it has been verified that the model is in agreements with the impedance of...
the variable speed drive. To do that, the resonances in the conducted frequency range (150 kHz–30 MHz) are measured and modeled (Table I). It can be seen that the simulated and measured resonances are close. The F2 resonance which is linked to the cable connecting the converter to the motor does not appear because it is not modeled in the simulation circuit.

TABLE I

<table>
<thead>
<tr>
<th>Measured and Simulated Resonance Frequencies</th>
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<td>Frequencies in MHz</td>
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<tr>
<td>Measured</td>
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<td>Simulated</td>
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C. Near Magnetic Field Studies

Then, a second analysis of the emitted near magnetic field, just over the static converter, is achieved. The Fourier transform of source currents and voltages, which are obtained by a time-domain simulation, are introduced as sources in our model. Then current distribution is solved and the magnetic field emitted is computed at a given frequency by means of the Biot and Savart law: the magnetic field $\mathbf{B}$ created by the NbVol inductive volume mesh elements of current density $\mathbf{j}$ is

$$
\mathbf{B}(r) = \frac{\mu_0}{4\pi} \sum_{l=1}^{\text{NbVol}} \int_{V_l} \int \mathbf{G}'(r, r_l) \cdot \mathbf{j}_l dV_l,
$$

(14)

The comparison between the vertical coordinate of the magnetic field Hz at 32-kHz simulated and measured is presented in Fig. 6. A good agreement between the two shades can be appreciated. It means that the main current loops are correctly modeled. These good results validate the methodology of all the modeling chain used to model this industrial device.

VI. CONCLUSION

In this paper, the coupling between two adapted integral methods has been presented. From two adapted meshings allowing, respectively, resistive-inductive and capacitive equivalent elements of power interconnections of industrial complexity have been extracted. The use of an adapted FMM to compute parasitic capacitances and the reduction technique of the RLM matrix into a smaller macroblock permits to deal with large and complex geometries with relatively few memory consumption.

It has also been highlighted that the complete (R-L-M-C) equivalent circuit can be exported into a SPICE-like tool where time-domain analysis can be performed. Then, the Fourier transforms of the currents and voltages obtained have been introduced as sources in the 3-D geometric representation of the interconnections in order to analyze the EMC performances of the system on a wide frequency range. The electrical behavior, the harmonic response and the emitted near magnetic field simulated compared to those obtained with measurements have validated this system-coupling approach on a real industrial variable speed drive.

ACKNOWLEDGMENT

The authors would like to thank J. Ecrabey from Schneider Electric and P. Loizelet from STIE for lending the variable speed drive and also O. Aouine and C. Labarre from ENSM for the near magnetic field measurement. In addition, the authors acknowledge P. Labie for the helpful programming around the FMM.

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